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Removal of the Resolvent-like Dependence on the Spectral Parameter from Perturbations[a]

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The spectral problem $(A+V(z))\psi = z\psi$ is considered with A, a self-adjoint operator. The perturbation $V(z)$ is assumed to depend on the spectral parameter z as resolvent of another self-adjoint operator A' : $V(z) = -B(A'-z)^{-1}B^*$. It is supposed that the operator B has a finite Hilbert-Schmidt norm and spectra of the operators A and A' are separated. Conditions are formulated when the perturbation $V(z)$ may be replaced with a "potential" W independent of z and such that the operator $H = A+W$ has the same spectrum and the same eigenfunctions (more precisely, a part of spectrum and a respective part of eigenfunctions system) as the initial spectral problem. The operator H is constructed as a solution of the non–linear operator equation $H = A + V(H)$ with a specially chosen operator–valued function $V(H)$. In the case if the initial spectral problem corresponds to a two–channel variant of the Friedrichs model, a basis property of the eigenfunction system of the operator H is proved. A scattering theory is developed for H in the case where the operator A has continuous spectrum.

1. Introduction

Perturbations, depending on the spectral parameter (usually energy of system) arise in a lot of quantum–mechanical problems typically (see e.g. Ref. [1]) as a result of dividing the Hilbert space $\mathcal{H}$ of physical system in two subspaces, $\mathcal{H} = \mathcal{H}_1 \oplus \mathcal{H}_2$. The first one, say $\mathcal{H}_1$, is interpreted as a space of some "external" degrees of freedom. The second one, $\mathcal{H}_2$, is associated with an "internal" structure of the system. The Hamiltonian $\mathbf{H}$ of the system looks as a matrix,

$$\mathbf{H} = \begin{bmatrix} A_1 & B_{12} \\ B_{21} & A_2 \end{bmatrix} \tag{1}$$

with A_α, $\alpha = 1, 2$, the channel Hamiltonians (self-adjoint operators in $\mathcal{H}_\alpha$) and B_{12}, $B_{21} = B_{12}^*$, the coupling operators. Reducing the spectral problem $\mathbf{H}U = zU$, $U = \{u_1, u_2\}$ to the channel α only one gets the spectral problem

$$[A_\alpha + V_\alpha(z)]u_\alpha = zu_\alpha, \quad \alpha = 1, 2, \tag{2}$$

where the perturbation

$$V_\alpha(z) = -B_{\alpha\beta}(A_\beta - zI_\beta)^{-1}B_{\beta\alpha}, \quad \beta \neq \alpha, \tag{3}$$

depends on the spectral parameter z as the resolvent $(A_\beta - zI_\beta)^{-1}$ of the Hamiltonian A_β. Here, by I_β we understand the identity operator in $\mathcal{H}_\beta$.

The present paper is a summary of the author's works [2]—[4] considering a possibility to "remove" the energy dependence from perturbations of the type (3). Namely, in [2]—[4] we search for such a new perturbation ("potential") W_α not depending on z that spectrum of the Hamiltonian $H_\alpha = A_\alpha + W_\alpha$ is (a part of) the spectrum of the problem (2). At the same time, the respective eigenvectors of H_α become also those for (2). An interest to the problem of such a removal of dependence on the spectral parameter from perturbations is stimulated in particular by a rather conceptual question (see for instance Ref. [5]) concerning a use of the two–body energy–dependent potentials in few–body nonrelativistic scattering problems. Since the energies of pair subsystems are not fixed in the N–body (N≥ 3) system, a direct embedding of such potentials into the few–body Hamiltonian is impossible. Thus, the replacements of the type (3) energy–dependent potentials with the respective new potentials W_α could be considered as a way to overcome this difficulty (see Ref. [4] for discussion).

The Hamiltonians H_α are found in [2]—[4] as solutions of the non-linear operator equations (first appeared in

Ref. [6])

$$H_\alpha = A_\alpha + V_\alpha(H_\alpha). \tag{4}$$

The operator-value function $V_\alpha(Y)$ of the operator variable Y, $Y : \mathcal{H}_\alpha \to \mathcal{H}_\alpha$, is defined by us in such a way [see formula (5)] that eigenfunctions ψ of Y, $Y\psi = z\psi$, become automatically those for $V_\alpha(Y)$ and $V_\alpha(Y)\psi = V_\alpha(z)\psi$. We have proved a solvability of Eq. (4) in the case where the Hilbert–Schmidt norm $\|B_{\alpha\beta}\|_2$ of the operators $B_{\alpha\beta}$ satisfies the condition $\|B_{\alpha\beta}\|_2 < \frac{1}{2}\mathrm{dist}\{\sigma(A_1), \sigma(A_2)\}$ in supposition that spectra $\sigma(A_\alpha)$ of the operators A_α are separated, $\mathrm{dist}\{\sigma(A_1), \sigma(A_2)\} > 0$ (see Theorem 1).

In Ref. [2], the problem of the removal of energy dependence from the type (3) perturbations was considered in details when one of the operators A_α is the Schrödinger operator in $L_2(\mathbb{R}^n)$ and another one has a discrete spectrum only. The report [3] announces the results concerning the equations (4) and properties of their solutions H_α in a rather more general situation where the Hamiltonian $\mathbf{H}$ may be rewritten in terms of a two-channel variant of the Friedrichs model investigated by O.A.Ladyzhenskaya and L.D.Faddeev [7]. In particular in [2] and [3] a scattering problem is studied for H_α in the case if A_α has continuous spectrum and the basis property of the eigenfunction system of the operator H_α is shown.

In the paper [4], we specify the assertions from [3] and give proofs for them. Also, we pay attention to an important circumstance disclosing a nature of solutions of the basic equations (4). Thing is that the operators $W_\alpha = V_\alpha(H_\alpha)$ may be present in the form $W_\alpha = B_{\alpha\beta}Q_{\beta\alpha}$ with $Q_{\beta\alpha}$ satisfying the stationary Riccati equations (7). Exactly the same equations always arise if one makes a block diagonalization of the type (1) operator matrices in the way described below in Lemma 1, so that the solutions H_α, $\alpha = 1, 2$, of Eqs. (4) determine in fact parts of the operator $\mathbf{H}$ in respective invariant subspaces. The idea of such a diagonalization was applied already by S.Okubo [8] to some quantum–mechanical Hamiltonians. It was used later by V.A.Malyshev and R.A.Minlos [9] in a method of construction of invariant subspaces for a class of self–adjoint operators in statistical physics. This idea was used also in the recent paper [10] by V.M.Adamjan and H.Langer who studied spectral properties of a class of the type (2) spectral problems and in particular, a possibility to choose among their solutions a Riesz basis in $\mathcal{H}_\alpha$.

2. Construction of the operators H_α

We study the spectral problem (2) with perturbation $V_\alpha(z)$ given by (3). We suppose that $B_{\beta\alpha}$ is a linear operator from $\mathcal{H}_\alpha$ to $\mathcal{H}_\beta$ with a finite Hilbert–Schmidt norm $\|B_{\beta\alpha}\|_2$, $\|B_{\beta\alpha}\|_2 < \infty$. A goal of the work is a construction of such an operator H_α that its each eigenfunction $u_\alpha, H_\alpha u_\alpha = z u_\alpha$, together with eigenvalue z, satisfies Eq. (2). The operator H_α is searched for as a solution of the non-linear operator equation (4). To obtain this equation we introduce the following operator-value function $V_\alpha(Y)$ of the operator variable Y :

$$V_\alpha(Y) = B_{\alpha\beta} \int\limits_{\sigma_\beta} E_\beta(d\mu) B_{\beta\alpha}(Y - \mu I_\alpha)^{-1}, \tag{5}$$

$Y : \mathcal{H}_\alpha \to \mathcal{H}_\alpha$. Here, σ_β is the spectrum and E_β, the spectral measure of the operator A_β. The integral over E_β in (5) for Y such that $\sup\limits_{\mu\in\sigma_\beta} \|(Y - \mu I_\alpha)^{-1}\| < \infty$ may be constructed in the same way as the usual integrals of scalar functions over spectral measure. For $\|B_{\beta\alpha}\|_2 < \infty$, the existence of this integral as a bounded operator from $\mathcal{H}_\alpha$ to $\mathcal{H}_\beta$ is proved in [4]. We notice that if ϕ is an eigenfunction of Y, $Y\phi = z\phi$, then automatically $V_\alpha(Y)\phi = B_{\alpha\beta} \int\limits_{\sigma_\beta} E_\beta(d\mu) B_{\beta\alpha}(z - \mu)^{-1}\phi = B_{\alpha\beta}(z - A_\beta)^{-1} B_{\beta\alpha}\phi = V_\alpha(z)\phi$. This means that H_α satisfies with its eigenfunctions ψ_α the relation $H_\alpha\psi_\alpha = (A_\alpha + V_\alpha(H_\alpha))\psi_\alpha$ and one can spread this relation over all the linear combinations of the eigenfunctions. Supposing that the eigenfunctions system of H_α is dense in $\mathcal{H}_\alpha$ one spreads this equation over all the domain $\mathcal{D}(A_\alpha)$. As a result we come to the desired *basic equation* (4) for H_α (see also [2]—[4] and Refs. therein). Eq. (4) means that the construction of the operator H_α is reduced to the searching for the operator $Q_{\beta\alpha} = \int\limits_{\sigma_\beta} E_\beta(d\mu) B_{\beta\alpha}(H_\alpha - \mu I_\alpha)^{-1}$. Since $H_\alpha = A_\alpha + B_{\alpha\beta}Q_{\beta\alpha}$, we have

$$Q_{\beta\alpha} = \int\limits_{\sigma_\beta} E_\beta(d\mu) B_{\beta\alpha}(A_\alpha + B_{\alpha\beta}Q_{\beta\alpha} - \mu I_\alpha)^{-1}, \quad \beta \neq \alpha. \tag{6}$$

We restrict ourselves to a study of Eq. (6) solvability only in the case where spectra σ_1 and σ_2 are separated, $d_0 = \mathrm{dist}(\sigma_1, \sigma_2) > 0$. Applying to Eq. (6) the contracting mapping theorem, one comes to the following:

T h e o r e m 1. *Let $M_{\beta\alpha}(\delta)$ be a set of bounded operators X, $X : \mathcal{H}_\alpha \to \mathcal{H}_\beta$, satisfying the inequality $\|X\| \leq \delta$*

with $\delta > 0$. If this δ and the norm $\|B_{\alpha\beta}\|_2$ satisfy the condition $\|B_{\alpha\beta}\|_2 < d_0 \min\{\frac{1}{1+\delta}, \frac{\delta}{1+\delta^2}\}$, then Eq. (6) is uniquely solvable in $M_{\beta\alpha}(\delta)$. In particular the equation (6) is uniquely solvable in the unit ball $M_{\beta\alpha}(1)$ for any $B_{\alpha\beta}$ such that $\|B_{\alpha\beta}\|_2 < \frac{1}{2}d_0$.

Eq. (6) can be rewritten (see [3], [4]) also in symmetric form as a stationary Riccati equation,

$$Q_{\beta\alpha}A_\alpha - A_\beta Q_{\beta\alpha} + Q_{\beta\alpha}B_{\alpha\beta}Q_{\beta\alpha} = B_{\beta\alpha}. \tag{7}$$

One finds immediately from Eqs. (7), $\alpha = 1, 2$, that if $Q_{\beta\alpha}$ gives a solution $H_\alpha = A_\alpha + B_{\alpha\beta}Q_{\beta\alpha}$ of the problem (4) in the channel α then $Q_{\alpha\beta} = -Q^*_{\beta\alpha} = -\int_{\sigma_\alpha}(H^*_\alpha - \mu I_\alpha)^{-1}B_{\alpha\beta}E_\beta(d\mu)$ gives an analogous solution $H_\beta = A_\beta + B_{\beta\alpha}Q_{\alpha\beta}$ in the channel β.

L e m m a 1. *Let $Q_{\beta\alpha}$ and $Q_{\alpha\beta} = -Q^*_{\beta\alpha}$ be solutions of Eqs. (7). Then the transform $\mathbf{H}' = \mathcal{Q}^{-1}\mathbf{H}\mathcal{Q}$ with $\mathcal{Q} = \begin{bmatrix} I_1 & Q_{12} \\ Q_{21} & I_2 \end{bmatrix}$ reduces the operator $\mathbf{H}$ to the block–diagonal form, $\mathbf{H}' = \mathrm{diag}\{H_1, H_2\}$ with $H_\alpha = A_\alpha + B_{\alpha\beta}Q_{\beta\alpha}$.*

One can find assertions analogous to Lemma 1 in Refs. [9] and [10]. A solvability (for sufficiently small $\|B_{\alpha\beta}\|$) of the equation (7) was proved in [9], [10] for different situations and by rather different methods but also in the supposition $\mathrm{dist}\{\sigma(A_1), \sigma(A_2)\} > 0$.

R e m a r k 1. Let $X_\alpha = I_\alpha - Q_{\alpha\beta}Q_{\beta\alpha} = I_\alpha + Q_{\alpha\beta}Q^*_{\alpha\beta}$. It follows from Lemma 1 that the operator $\tilde{\mathcal{Q}} = \mathcal{Q}X^{-1/2}$ with $X = \mathrm{diag}\{X_1, X_2\}$ is unitary. Thus, the operator $\mathbf{H}'' = \tilde{\mathcal{Q}}^*\mathbf{H}\tilde{\mathcal{Q}} = X^{1/2}\mathbf{H}'X^{-1/2}$ becomes self-adjoint in $\mathcal{H}$. Since $\mathbf{H}'' = \mathrm{diag}\{H''_1, H''_1\}$ with $H''_\alpha = X^{1/2}_\alpha H_\alpha X^{-1/2}_\alpha$, the operators H''_α, $\alpha = 1, 2$, are self-adjoint on $\mathcal{D}(A_\alpha)$ in $\mathcal{H}_\alpha$. Moreover, the operators $\mathbf{H}^{(\alpha)} = \tilde{\mathcal{Q}} \cdot \mathrm{diag}\{H''_\alpha, 0\} \cdot \tilde{\mathcal{Q}}^* = \mathcal{Q} \cdot \mathrm{diag}\{H_\alpha, 0\} \cdot \mathcal{Q}^{-1}$ represent parts of the Hamiltonian $\mathbf{H}$ in the corresponding invariant subspaces $\mathcal{H}^{(\alpha)} = \{f : \ f = \{f_\alpha, f_\beta\} \in \mathcal{H}, \ f_\alpha \in \mathcal{H}_\alpha, \ f_\beta = Q_{\beta\alpha}f_\alpha\}$ (see also Refs. [9], [10]).

3. Spectra of the Hamiltonians H_α and basis properties of their eigenfunctions

Let us suppose that $Q_{\beta\alpha}$ and $Q_{\alpha\beta} = -Q^*_{\beta\alpha}$ are solutions of Eqs. (6) and (7) which are spoken about in Theorem 1. Since we take $B_{\alpha\beta}$ with $\|B_{\alpha\beta}\| \le \|B_{\alpha\beta}\|_2 < d_0/2$ and $\|Q_{\beta\alpha}\| < 1$, the spectra $\sigma(H_1)$ and $\sigma(H_2)$ do not intersect (actually, when these spectra are discussed in Refs. [3], [4], a more general case is also considered where not necessary $\|Q_{\beta\alpha}\| < 1$). By Lemma 1, the operator $\mathbf{H}' = \mathrm{diag}\{H_1, H_2\}$ is connected with the (self-adjoint) operator $\mathbf{H}$ by a similarity transform. Thus, the spectra $\sigma(H_1)$ and $\sigma(H_2)$ of the operators H_α, $\alpha = 1, 2$, are real and $\sigma(H_1)\bigcup\sigma(H_2) = \sigma(\mathbf{H})$. Continuous spectrum $\sigma_c(H_\alpha)$ of every H_α coincides with that, σ^c_α, of the operator A_α, $\sigma_c(H_\alpha) = \sigma^c_\alpha$, since due to $\|B_{\alpha\beta}\|_2 < +\infty$, the potential $W_\alpha = B_{\alpha\beta}Q_{\beta\alpha}$ is a compact operator.

For more concrete statements concerning the spectra of the operators H_α we accept some presuppositions restricting us as regards $\mathbf{H}$ to the case of a two–channel variant of the Friedrichs model in the form [7] reproducing often encountered quantum–mechanical situations. At first, we assume that the operator $\mathbf{H}$ is defined in that representation where the operators A_α, $\alpha = 1, 2$, are diagonal. We suppose that the continuous spectra σ^c_α are absolutely continuous and consist of a finite number of finite (and may be one or two infinite) intervals. At second, we suppose that discrete spectra σ^d_α of the operators A_α, $\alpha = 1, 2$, do not intersect with σ^c_α, $\sigma^d_\alpha\bigcap\sigma^c_\alpha = \emptyset$, and consist of a finite number of points with finite multiplicity. The coupling operators $B_{\alpha\beta}$ are supposed to be the integral ones with sufficiently quickly decreasing (in the case of unbounded σ^c_α) kernels being smooth in the Hölder sense (see Refs. [3], [4] for details).

With these presuppositions the continuous spectrum $\sigma_c(\mathbf{H}) = \sigma^c_1 \cup \sigma^c_2$ of the operator $\mathbf{H}$ is absolutely continuous and its part $\mathbf{H}^c$ acting in respective invariant subspace, is unitary equivalent to the operator $\mathbf{H}_0 = A^{(0)}_1 \oplus A^{(0)}_2$ with $A^{(0)}_\alpha$, $\alpha = 1, 2$, the part of A_α acting in the invariant subspace $\mathcal{H}^c_\alpha$ corresponding to σ^c_α. Namely, there exist the wave operators $U^{(+)}$ and $U^{(-)}$, $U^{(\pm)} = \begin{pmatrix} u^{(\pm)}_{11} & u^{(\pm)}_{12} \\ u^{(\pm)}_{21} & u^{(\pm)}_{22} \end{pmatrix} = s - \lim\limits_{t\to\mp\infty} \mathrm{e}^{i\mathbf{H}t}\mathrm{e}^{-i\mathbf{H}_0 t}$, with the properties: $\mathbf{H}U^{(\pm)} = U^{(\pm)}\mathbf{H}_0$, $U^{(\pm)*}U^{(\pm)} = I$, $U^{(\pm)}U^{(\pm)*} = I - P_d$. Here, by P_d we understand the orthogonal projector on the subspace corresponding to the discrete spectrum $\sigma_d(\mathbf{H})$ of $\mathbf{H}$. The kernel $u^{(\pm)}_{\alpha\alpha}(\lambda, \lambda')$ of the component $u^{(\pm)}_{\alpha\alpha}$, $\alpha = 1, 2$, represents a (generalized) eigenfunction of continuous spectrum of the problem (2) for $z = \lambda' \pm i0$, $\lambda' \in \sigma^c_\alpha$. At the same time $u^{(\pm)}_{\alpha\beta}(\lambda, \lambda')$ is the problem (2) eigenfunction corresponding to $\lambda' \in \sigma^c_\beta$.

By U_j, $j = 1, 2, \ldots$, we denote eigenvectors, $U_j = \{u^{(j)}_1, u^{(j)}_2\}$, $\|U_j\| = 1$, and by z_j, $z_j \in \mathbb{R}$, the respective eigenvalues of $\sigma_d(\mathbf{H})$. We assume that in the case of multiple discrete eigenvalues, certain z_j may be repeated in the

numeration. The component $u_\alpha^{(j)}$ of the vector U_j is a solution of Eq. (2) for $z = z_j$.

Let us return, with the presuppositions above, to the operators H_α. First, let us assume $\sigma_d(H_\alpha) \neq \emptyset$. Then, it follows from the construction of the function (5) that if $z \in \sigma_d(H_\alpha)$ then this z becomes automatically a point of the discrete spectrum of the initial spectral problem (2). At the same time ψ_α becomes its eigenfunction. We shall denote the eigenfunctions of the H_α by $\psi_\alpha^{(j)}$, $\psi_\alpha^{(j)} = u_\alpha^{(j)}$, keeping for them the same numeration as for the eigenvectors U_j, $U_j = \{u_\alpha^{(j)}, u_\beta^{(j)}\}$, of the Hamiltonian $\mathbf{H}$, $\mathbf{H}U_j = z_j U_j$, $z_j \in \sigma_d(\mathbf{H})$. Respective eigenvectors of the adjoint operator H_α^*, $H_\alpha^* = A_\alpha + Q_{\beta\alpha}^* B_{\beta\alpha}$, are $\tilde{\psi}_\alpha^{(j)} = \psi_\alpha^{(j)} - Q_{\alpha\beta} u_\beta^{(j)}$. Due to Lemma 1, $\sigma_d(\mathbf{H}) = \sigma_d(H_1) \bigcup \sigma_d(H_2)$. Since in conditions of Theorem 1 $\sigma(H_1) \bigcap \sigma(H_2) = \emptyset$, we have also $\sigma_d(H_1) \bigcap \sigma_d(H_2) = \emptyset$.

Dealing with the continuous spectrum of H_α we take into account the fact that the solutions $Q_{\beta\alpha}$ and $Q_{\alpha\beta} = -Q_{\beta\alpha}^*$ of Eqs. (6) and (7) corresponding to the operators $B_{\alpha\beta}$ with Hölder kernels, have the Hölder kernels themselves. The same is true as well for $W_\alpha = B_{\alpha\beta} Q_{\beta\alpha}$. Then we can prove [3], [4] that the operators $\Psi_\alpha^{(\pm)} = u_{\alpha\alpha}^{(\pm)}$ turn out to be the wave operators between H_α and $A_\alpha^{(0)}$: $\Psi_\alpha^{(\pm)} = s-\lim\limits_{t\to\mp\infty} \exp(iH_\alpha t)\exp(-iA_\alpha^{(0)} t)$ and $H_\alpha \Psi^{(\pm)} = \Psi_\alpha^{(\pm)} A_\alpha^{(0)}$. At the same time the operators $\tilde{\Psi}_\alpha^{(\pm)} = \Psi_\alpha^{(\pm)} - Q_{\alpha\beta} u_{\beta\alpha}^{(\pm)}$ become those ones for H_α^*.

T h e o r e m 2. *The following orthogonality relations take place:* $\langle \psi_\alpha^{(j)}, \tilde{\psi}_\alpha^{(k)} \rangle = \delta_{jk}$, $\Psi_\alpha^{(\pm)*} \tilde{\Psi}_\alpha^{(\pm)} = I_\alpha|_{\mathcal{H}_\alpha^c}$, $\tilde{\Psi}_\alpha^{(\pm)*} \psi_\alpha^{(j)} = 0$ *and* $\Psi_\alpha^{(\pm)*} \tilde{\psi}_\alpha^{(j)} = 0$. *Also, the completeness relations are valid,* $\sum\limits_{j: H_\alpha u_\alpha^{(j)} = z_j u_\alpha^{(j)}} \psi_\alpha^{(j)} \langle \cdot, \tilde{\psi}_\alpha^{(j)} \rangle + \Psi_\alpha^{(\pm)} \tilde{\Psi}_\alpha^{(\pm)*} = I_\alpha$, $\alpha = 1, 2$. *For all this* $S^{(\alpha)} = \Psi_\alpha^{(-)-1} \Psi_\alpha^{(+)} = \tilde{\Psi}_\alpha^{(-)*} \Psi_\alpha^{(+)} = \Psi_\alpha^{(-)*} X_\alpha \Psi_\alpha^{(+)}$ *represents a scattering operator for a system described by the Hamiltonian* H_α. *In fact, this operator coincides with the component* $S_{\alpha\alpha}$ *of the scattering operator* S, $S = U^{(-)*} U^{(+)}$, *for a system described by the two–channel Hamiltonian* $\mathbf{H}$.

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